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DETERMINATION OF OPTIMUM SAMPLING INTERVAL IN DEM DATA ACQUISITION

A Guide for Users

by

Alberta

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OPTIMUM SAMPLING INTERVAL
IN DEM DATA ACQUISITION

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by

A.E. BALCE, P. ENG., C.L.S.

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1. INTRODUCTION

This guide provides guidelines for determining the optimum sampling interval(s) in data acquisition of digital elevation models (DEM) by grid sampling using photogrammetric methods. Given one or more sample profiles from a stereomodel and the error budget σ_{int} for height interpolation from the DEM data, program RF is run to determine the roughness factor of each sample profile. Knowing the roughness factor, length and sampling rate of a profile, the most suitable program is chosen to compute the optimum sampling interval. Programs SPECTRA3, LOGKV3, LINEAR1 and RF are available for this purpose. Guidelines for using these programs are based upon the test results described in Appendices 1 and 2. These programs are coded in FORTRAN and are operational on the Bureau's VAX 11/785.

Program source code, sample data and user guide are available from:

Tony Balce, Section Head
Photogrammetric Control
Survey Control Branch
Alberta Bureau of Surveying & Mapping
4949 - 94B Avenue
Edmonton, Alberta T6B 2T5

Telephone: (403)427-3138

Users are requested to provide their own magnetic tapes for the program source code and sample data. They will be invoiced for tape handling and processing.

References cited in this guide are listed in Appendix 1.

2. DESCRIPTION OF PROGRAMS

2.1 Program SPECTRA3

This program was made available by Dr. D. Fritsch, Chair of Photogrammetry, Technical University of Munich, Federal Republic of Germany (Fritsch, 1984).

Given a sample profile $Z(i)$, $i = 0,1,2, \ldots N-1$ measured at a fixed horizontal spacing Δd, its linear trend is first removed as follows:

$$\bar{Z}(i) = Z(i) - a_0 - a_1 i, \quad i = 0,1,2, \ldots N-1 \tag{1}$$

where a_0 and a_1 are parameters of a linear function which fits best

the given profile obtained by least squares. The residuals $\bar{Z}(i)$ are then represented by the inverse discrete Fourier transform and the profile reconstructed as follows (Fritsch, 1985):

$$Z_r(i) = \frac{1}{N} \sum_{k=0}^{R<N/2-1} X(k) e^{j\pi ki/N}, \quad j = \sqrt{-1} \tag{2}$$

$$X(k) = \sum_{i=0}^{N-1} \bar{Z}(i) e^{-j\pi ki/N} \tag{3}$$

The variance of the reconstruction process is

$$\sigma_r^2 = \frac{1}{N} \sum_{i=0}^{N-1} \{\bar{Z}(i) - Z_r(i)\}^2 \tag{4}$$

which must not exceed σ_{int}^2. The above computations are done iteratively to determine the cut-off frequency $\omega(R) = 2\pi R/N$. Hence, the optimum sampling interval is:

$$\Delta d_{opt} = \frac{\pi}{\omega(R)} (\Delta d) \tag{5}$$

2.2 Program LOGKV3

The underlying concept of this program is self-similarity which was investigated and implemented by Dr. K. Kubik, Department of Geodetic Science and Surveying, The Ohio State University, Columbus, Ohio (Frederiksen, Jacobi and Kubik, 1983 and 1984).

Given a sample profile $Z(i)$, $i = 1,2, \ldots N$ measured at a fixed horizontal spacing Δd, its linear trend is first removed as with the program SPECTRA:

$$\bar{Z}(i) = Z(i) - a_0 - a_1 i, \quad i = 1,2, \ldots N. \tag{6}$$

From the residuals $\bar{Z}(i)$, the variances of height differences at varying lags k are computed:

$$V(k) = \frac{1}{N-k} \sum_{i=1}^{N-k} \{\bar{Z}(i) - \bar{Z}(i+k)\}^2, \quad k = 1,2,\ldots N/2 \tag{7}$$

Figure 1 of Appendix 1 represents the plot of natural logarithms of V(k) versus k, also known as the log-log plot of variogram modeled as

$V(k) = ck^{\beta}$. Based on computations with real data, the points tend to lose correlation at larger values of k (see points 10 to 16). Based upon the first few points (1 to 9) that are well correlated, a straight line may be drawn, from which slope β and the ordinate-intercept ln(c) are determined graphically.

β and ln(c) may also be determined analytically. Starting with the first three points ln(V(k)), k = 1,2,3, a straight line function is determined by least squares. Residuals are computed and compared with a threshold that is equivalent to an acceptable graphical error. If residuals are less than the given threshold, then the process is repeated with the next point included, until at least one residual exceeds the given threshold. This is the way β and ln(c) are determined by this program. Hence, manual plotting is completely avoided.

The optimum sampling interval is

$$\Delta d_{opt} = L\Delta d \tag{8}$$

where L is computed from

$$\frac{\sigma^2_{int}}{c} = (L^{\beta})\left(\frac{2-2^{\beta-1}}{2^{\beta+1}}\right) \tag{9}$$

2.3 Program LINEAR1

This program is based on the assumption that the DEM data are sufficient in density such that heights can be interpolated accurately by a straight line or planar function.

Given a sample profile Z(i), i = 1,2,...N measured at a fixed horizontal spacing Δd, the optimum sampling interval Δd_{opt} is determined iteratively. Starting with a control spacing of 2Δd, intermediate sample points are interpolated linearly and elevations compared with their respective measured elevations. The root mean square (RMS) value of the discrepancies is computed and compared with σ_{int}. If RMS is less than σ_{int}, the process is repeated by increasing the control spacing by Δd. If kΔd is the control spacing for each iteration, k = 2,3,...K where K is the least value of k whereby σ_{int} is exceeded, then

$$\Delta d_{opt} = (K-1 + \varepsilon)\Delta d \qquad (10)$$

where

$$\varepsilon = \frac{\sigma_{int} - RMS(K-1)}{RMS(K) - RMS(K-1)} \qquad (11)$$

When $K = 2$, $\Delta d_{opt} = \Delta d$, assuming $RMS(1) = \sigma_{int}$.

2.4 Program RF

This program was developed primarily for analyzing terrain undulation based on the following parameters (Frederiksen, Jacobi and Kubik, 1984):

a) Relief is the height difference between the lowest and highest elevations of a given profile.

b) Slope is the first derivative of the profile at a point.

c) Wavelength is the average distance between successive points of zero slope (e.g. two hilltops).

In this program:

a) Relief is the average height difference Δz_{ave} between successive significant breakpoints.

b) Wavelength is the average distance, Δd_{ave} between successive significant breakpoints. The optimum sampling interval is

$$\Delta d_{opt} = (\Delta d_{ave})/2 \qquad (12)$$

c) $\text{Mean slope} = (\Delta z_{ave}/\Delta d_{ave})\ 100\% \qquad (13)$

Significant breakpoints are determined using the following criteria:

a) Height difference between any successive points exceeds σ_{int}.

b) If a point is linearly interpolated between its two neighboring points, and its computed elevation is compared with its sampled elevation, the discrepancy exceeds σ_{int}.

The roughness factor, R.F. in this program is defined to be the mean slope (Equation 13).

3. INPUT DESCRIPTION

All four programs are run using the same free format of the data as follows:

```
                        PROFILE 200-1 <==-------------------- Item   1)
2.1  25  <==------------------------------------------------------  2)
200        278081.138 5459645.873 1595.666 36 <==--┐
200        278080.122 5459670.853 1591.156 37      |
200        278079.106 5459695.838 1587.881 37      |
200        278078.090 5459720.810 1584.791 37      |
200        278077.074 5459745.791 1583.061 37      |
200        278076.058 5459770.775 1569.715 37      |
200        278075.042 5459795.748 1569.344 37      |                3)
200        278074.026 5459820.732 1578.674 37      |
200        277971.410 5462343.631 2006.622 37      |
200        277970.394 5462368.610 1995.994 37      |
200        277969.378 5462393.590 1980.732 37      |
200        277968.362 5462418.569 1964.667 37 <==--┘
  0                 0           0 100000  <==------                 4)
           PROFILE 200-2
 2.1 25
200        277967.346 5462443.548 1949.962 37
200        277966.330 5462468.527 1935.194 37
200        277965.314 5462493.507 1918.326 37
```

Item 1) Profile identifier

2) σ_{int}, Δd

3) Profile data: Point number, X, Y, Z

4) End of profile indicator: Z> 100 000

4. OUTPUT DESCRIPTIONS

Linear units of the output are the same as those of the input.

4.1 SPECTRA3 Output

```
**************************************************

                     PROFILE 202

SIGMA=      2.100 PROFILE DENSITY=      25.00

K=   4.923077

LEAST SAMPLING DENSITY=     123.08

 SIGMA OF RECONSTRUCTION=       2.0

**************************************************

                     PROFILE 204

SIGMA=      2.100 PROFILE DENSITY=      25.00

K=   8.000000

LEAST SAMPLING DENSITY=     200.00

 SIGMA OF RECONSTRUCTION=       2.1

**************************************************
```

The information presented for profile 202 are:

a) Profile identifier = PROFILE 202

b) $\sigma_{int} = 2.100$

c) $\Delta d = 25.00$

d) $\pi/\omega(R) = 4.923077$

e) $\Delta d_{opt} = 123.08$

f) RMS of profile reconstruction = 2.0

4.2 LOGKV3 Output

```
0***************************************************
0                     PROFILE 200
0SIGMA=     2.100 PROFILE DENSITY=     25.00
0RMS=      0.01 RESMAX=     -0.01 C=      3.888 BETA=  1.885500 K=   3
0RMS=      0.01 RESMAX=     -0.01 C=      3.893 BETA=  1.870306 K=   4
0RMS=      0.02 RESMAX=      0.02 C=      3.900 BETA=  1.854522 K=   5
0RMS=      0.02 RESMAX=      0.03 C=      3.910 BETA=  1.838513 K=   6
0RMS=      0.03 RESMAX=      0.04 C=      3.920 BETA=  1.822787 K=   7
0RMS=      0.03 RESMAX=      0.05 C=      3.932 BETA=  1.807393 K=   8
0OPTIMUM DENSITY=     40.79  L=  1.631455
0***************************************************
0                     PROFILE 202
0SIGMA=     2.100 PROFILE DENSITY=     25.00
0RMS=      0.01 RESMAX=     -0.01 C=      2.011 BETA=  1.736785 K=   3
0RMS=      0.01 RESMAX=     -0.02 C=      2.017 BETA=  1.718779 K=   4
0RMS=      0.02 RESMAX=      0.02 C=      2.025 BETA=  1.702266 K=   5
0RMS=      0.02 RESMAX=      0.03 C=      2.034 BETA=  1.686904 K=   6
0RMS=      0.03 RESMAX=      0.04 C=      2.043 BETA=  1.672593 K=   7
0RMS=      0.03 RESMAX=      0.05 C=      2.054 BETA=  1.658360 K=   8
0RMS=      0.04 RESMAX=      0.06 C=      2.066 BETA=  1.644120 K=   9
0OPTIMUM DENSITY=     88.49  L=  3.539616
0***************************************************
```

The information presented for profile 200 are:

a) Profile identifier = PROFILE 200

b) σ_{int} = 2.100

c) Δd = 25.00

d) For the last iteration of linear regression:
   - RMS of residuals = 0.03
   - Maximum residual = 0.05
   - ln(c) = 3.932
   - β = 1.807393
   - total number of points used = 8

e) L = 1.631455

f) Δd_{opt} = 40.79

4.3 LINEAR1 Output

```
**************************************************

                    PROFILE 202

SIGMA=        2.10        PROFILE DENSITY=      25.00
K=      1
RMS=        1.07
K=      2
RMS=        1.54
K=      3
RMS=        1.82
K=      4
RMS=        2.46

OPTIMUM SAMPLING INTERVAL=    110.81

**************************************************

                    PROFILE 204

SIGMA=        2.10        PROFILE DENSITY=      25.00
K=      1
RMS=        0.98
K=      2
RMS=        1.31
K=      3
RMS=        1.72
K=      4
RMS=        1.69
K=      5
RMS=        2.28

OPTIMUM SAMPLING INTERVAL=    142.35

**************************************************
```

The information presented for profile 202 are:

a) Profile identifier = PROFILE 202

b) $\sigma_{int} = 2.10$

c) $\Delta d = 25.00$

d) RMS of discrepancies for the last iteration = 2.46

e) $\Delta d_{opt} = 110.81$

4.4 RF Output

```
*****************************

                  PROFILE 202

 SIGMA=       2.1 SAMPLING INTERVAL=     25.0 TOTAL NO. OF POINTS=      479

TOTAL NO. OF SIGNIFICANT BREAK POINTS=        77

RF=     6.922 AVEDH=     10.60 AVEDIS=   153.20
DHMIN=      2.10 DHMAX=     51.53 DISMIN=     24.99 DISMAX=    750.00

 OPTIMUM SAMPLING INTERVAL=      76.6

*****************************

                  PROFILE 204

 SIGMA=       2.1 SAMPLING INTERVAL=     25.0 TOTAL NO. OF POINTS=      457

TOTAL NO. OF SIGNIFICANT BREAK POINTS=        64

RF=     4.556 AVEDH=      7.99 AVEDIS=   175.38
DHMIN=      2.10 DHMAX=     25.46 DISMIN=     25.00 DISMAX=    650.00

 OPTIMUM SAMPLING INTERVAL=      87.7
```

The information presented for profile 202 are:

a) Profile identifier = PROFILE 202

b) σ_{int} = 2.1

c) Δd = 25.0

d) Total number of sample points = 479

e) Roughness factor = 6.922%

f) The following information pertains to successive significant breakpoints:
- Δz_{ave} = 10.60
- Δd_{ave} = 153.20
- minimum height difference = 2.10
- maximum height difference = 51.53
- minimum distance = 24.99
- maximum distance = 750.00

g) Δd_{opt} = 76.6

5. GUIDELINES FOR DEM SAMPLING

5.1 Profile Data Acquisition

a) Measure breaklines, hilltops and depressions, preferably plotted on the manuscript or shown on an interactive graphics display interfaced with the stereoplotter.

b) Decide whether or not the model will be partitioned into regions of varying grid sizes, depending on the variations of terrain undulation.

c) If the decision is made to partition the model, then delineate boundaries of regions and excluded areas (water bodies, cloud cover, etc.), preferably indicated on a manuscript or shown on an interactive graphics display.

d) Measure at least one sample profile at a fixed sample spacing, in each region or the entire model. Each profile should be representative of the terrain undulation. Avoid breaklines that have been measured previously. Each profile should be as long and as dense as possible for more reliable results.

5.2 Determination of σ_{int}

For more details, see Appendix of Appendix 1.

$$\sigma_{int} \approx (\sigma^2_{spec} - \sigma^2_{AT} - \sigma^2_{setup} - \sigma^2_{samp})^{1/2} \qquad (14)$$

where

$$\sigma_{spec} = \frac{CI}{2(1.64)} \text{ in metres} \qquad (15)$$

CI = contour interval in metres

$$\sigma^2_{AT} = RMS^2_{TP} + RMS^2_{CP} \qquad (16)$$

RMS_{TP} = RMS of vertical tie point residuals after block adjustment in metres.

RMS_{CP} = RMS of vertical ground control point residuals after block adjustment in metres

If no aerial triangulation has been done, such as in case of full-model control or leveling points transferred from other triangulated photography, then use the appropriate standard deviation for those points.

$$\sigma_{setup} \approx \sigma_{samp} = \frac{H}{CF(2)(1.64)} \text{ in metres} \qquad (17)$$

H = flying height above mean ground elevation in metres
CF = C-factor (see Table 1)

For large-scale photography, $\sigma_{setup} \approx \sigma_{samp}$ must be estimated as the RMS of vertical residuals at photogrammetric/ground control points after exterior orientation.

Table 1

C-FACTORS

(Thorpe 1984)

Instrument	C-Factors Commercial	C-Factors U.S. Federal Agencies
All analytical plotters	3000	2500
Zeiss Planimat	2400	2100
Wild A10	2400	2100
Wild Aviomap	2400	2100
Santoni IV	2400	2100
Kern PG3	2400	2100
Jena Stereometrograph	2400	2100
Wild AG1	2000	1800
Wild A8	2000	1800
Santoni III	2000	1800
Santoni IIC	2000	1800
Galileo G7	2000	1800
Galileo G6	2000	1800
Kern PG2	2000	1800
Zeiss Planitop	1800	1500
Wild B8	1800	1500
Jena Topocart	1800	1500
Kelsh	1500	1200

5.3 Determination of Optimum Sampling Interval

a) Run program RF to determine the roughness factor for each profile.

b) From Tables 2 and 3, use the most appropriate program, given the photo scale and profile attributes. If more than one program is recommended for a set of profile attributes, the recommended programs are given in the order of increasing degreee of optimism, i.e., the first program is the most conservative, while the last is the most optimistic. In this way, the user has the flexibility of choosing the most appropriate program depending on each project situation. For example, if the user has a progressive sampling facility, the most appropriate program is the one that is most optimistic for computing the zero-order sampling interval. On the other hand if the operator is inexperienced and DEM data is to be acquired without progressive sampling facility, for large-scale application, the most appropriate program in this case is the one that is most conservative.

Low sampling rate with short profile must be avoided because sample points may be too few to produce reliable result. For large-scale application, flat and short profile, even with high sampling rate must also be avoided.

Table 2

SMALL-SCALE PROGRAM GUIDELINES (PHOTO SCALE < 1:10 000)

Profile Attributes	Program to Use
Rough, long, high	LOGKV3, LINEAR1, or RF
Rough, long, low	RF, LINEAR1, or SPECTRA3
Rough, short, high	RF, LINEAR1, LOGKV3 or SPECTRA3
Flat, long, high	LINEAR1, LOGKV3 or RF
Flat, long, low	LOGKV3, RF or LINEAR1
Flat, short, high	RF or LOGKV3

Notes on profile attributes:

- Rough means R.F. $\geq$ 15%
- Flat means R.F. < 15%
- Long means length $\geq$ 1.75 model base
- Short means length < 1.75 model base
- High means $\Delta d \leq 0.8$ mm at image scale
- Low means $\Delta d > 0.8$ mm at image scale.

TABLE 3

LARGE-SCALE PROGRAM GUIDELINES (PHOTO SCALE ⩾ 1:10 000)

Profile Attributes	Program to Use
Rough, long, high	LOGKV3, RF, SPECTRA3 or LINEAR1
Rough, long, low	RF
Rough, short, high	RF, or LOGKV3
Flat, long, high	RF, LOGKV3 or SPECTRA3
Flat, long, low	RF, LOGKV3, SPECTRA3 or LINEAR1

Notes on profile attributes:

- Rough means R.F. ⩾ 5%
- Flat means R.F. < 5%
- Long means length ⩾ 1.75 model base
- Short means length < 1.75 model base
- High means $\Delta d \leqslant 1.0$ mm at image scale
- Low means $\Delta d > 1.0$ mm at image scale.

c) Depending on the results, decide whether to use one or two sizes for grid sampling in the two perpendicular directions for each region or the entire model. If only one size is used, then use the mean sampling interval.

5.4 Grid Sampling and Contouring

a) Sample each region or the entire model using the final sampling interval(s), in stationary mode.

b) From the elevations of breaklines, hilltops, depressions and grid samples, generate contour lines for the entire model and plot them at the same scale as the manuscript on the plotting table of the stereoplotter or display them on an interactive graphics display.

c) Scan the model and check the contour lines for completeness, fidelity and accuracy.

d) While scanning the model, additional points may be sampled as required and contour lines regenerated. An example of an interactive graphics DEM sampling is presented in Ostman (1982).

APPENDIX 1

DETERMINATION OF OPTIMUM SAMPLING INTERVAL IN GRID DIGITAL ELEVATION MODELS (DEM) DATA ACQUISITION

(Presented Paper, Annual ASPRS/ACSM Convention,
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DETERMINATION OF OPTIMUM SAMPLING INTERVAL IN GRID DIGITAL ELEVATION MODELS (DEM) DATA ACQUISITION

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DETERMINATION OF OPTIMUM SAMPLING INTERVAL IN GRID DIGITAL ELEVATION MODELS (DEM) DATA ACQUISITION

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BIOGRAPHICAL SKETCH

A. E. Balce is the section head of Photogrammetric Control, Survey Control Branch, ABSM. He received his B.Sc. in geodetic engineering from the University of the Philipines, Quezon City in 1970; and M. Sc. in photogrammetric engineering from the International Institute for Aerial Survey and Earth Sciences (ITC), Enschede, The Netherlands in 1974. He is a registered professional engineer in Alberta and a commissioned Canada Lands Surveyor.

ABSTRACT

Four computer programs for estimating the optimum sampling interval in grid sampling of digital elevation models (DEM) by photogrammetric methods, have been tested with real data and results compared. These programs are SPECTRA using inverse discrete Fourier transform, LOGKV using self-similarity concept, and LINEAR and RF, both based on linear interpolation. They were tested with real sample profiles of three categories: long (≈12 km), short(≈3 km), both at fixed sample spacing of 25 m, and long (≈12 km) at varying sample spacings of 50, 75 and 100 m. These sample profiles were obtained using an analytical stereoplotter, from four contiguous stereomodels of aerial photographs at scale of 1:60 000. For the purpose of validating the results, one of the stereomodels was sampled three times at grid intervals of 75 and 130 m for the entire model, and 25 m for its southwest quarter portion. Together with the measurements of breaklines, hilltops and depressions, contour lines were interpolated and plotted from each grid sampling. Index contour lines from each plot were tested for accuracy on an analytical stereoplotter. From the results, guidelines for using these programs are presented and a sampling strategy is recommended.

INTRODUCTION

An optimum sampling strategy for data acquisition of DEM by photogrammetric methods, involves the following basic steps:

a) Selective sampling of abrupt changes in elevations such as breaklines, hilltops and depressions.
b) Grid sampling using regular, semi-regular or irregular sampling intervals.
c) Progressive sampling using second-height differences (Makarovic, 1977) or harmonic vector magnitude computed from double Fourier function (Ayeni, 1982).

The purpose of an optimum sampling strategy is to acquire DEM data with the least number of sample points and yet produce contour lines that meet the required accuracy specifications. The above strategy looks ideal, but actually only steps (a) and (b) are commonly implemented in practice. As a result the DEM data is either under- or over-sampled, with the latter prevailing mostly as a safety factor. Less optimum sampling, i.e. under- or over-sampling both have an economic implication. Under-sampling will result in inaccurate DEM data which may be rejected or require resampling. On the other hand, over-sampling will result in unnecessary observation time, extra computer data processing for redundant data and computer storage problem. Hence, it is imperative to implement an optimum sampling strategy.

This paper deals mainly with test comparisons of four computer programs for determining optimum sampling intervals in grid sampling given one or more profiles representing a stereomodel or its portion. The optimum sampling interval is determined by each program in such a way that the subsequent DEM data to be acquired (consisting of breaklines, hilltops, depressions and grid samples), meet the specified accuracy of interpolated heights (e.g. contour lines). Empirically, the error budget for the specified accuracy consists of accuracies of aerial triangulation, model setup, sampling and interpolation (see the Appendix). The only unknown in the error budget is the required accuracy for interpolation σ_{int}, which can be estimated readily knowing the other accuracy estimates. The required accuracy for interpolation is the key factor in determining the optimum sampling interval as computed by the four programs.

DESCRIPTION OF PROGRAMS

Program SPECTRA

This program was made available by Dr. D. Fritsch, Chair of Photogrammetry, Technical University of Munich, Federal Republic of Germany (Fritsch, 1984).

Given a sample profile $Z(i)$, $i = 0,1,2, \dots N-1$ measured at a fixed horizontal spacing Δd, its linear trend is first removed as follows:

$$\bar{Z}(i) = Z(i) - a_0 - a_1 i, \quad i = 0,1,2, \dots N-1 \tag{1}$$

where a_0 and a_1 are parameters of a linear function which fits best

the given profile obtained by least squares. The residuals $\bar{Z}(i)$ are then represented by the inverse discrete Fourier transform and the profile reconstructed as follows (Fritsch, 1985):

$$Z_r(i) = \frac{1}{N} \sum_{k=0}^{R<N/2-1} X(k) e^{j\pi k i/N}, \quad j = \sqrt{-1} \tag{2}$$

$$X(k) = \sum_{i=0}^{N-1} \bar{Z}(i) e^{-j\pi k i/N} \tag{3}$$

The variance of the reconstruction process is

$$\sigma_r^2 = \frac{1}{N} \sum_{i=0}^{N-1} \{\bar{Z}(i) - Z_r(i)\}^2 \tag{4}$$

which must not exceed σ_{int}^2. The above computations are done iteratively to determine the cut-off frequency $\omega(R) = 2\pi R/N$. Hence, the optimum sampling interval is:

$$\Delta d_{opt} = \frac{\pi}{\omega(R)} (\Delta d) \tag{5}$$

Program LOGKV

The underlying concept of this program is self-similarity which was investigated and implemented by Dr. K. Kubik, Department of Geodetic Science and Surveying, The Ohio State University, Columbus, Ohio (Frederiksen, Jacobi and Kubik, 1983 and 1984).

Given a sample profile $Z(i)$, $i = 1,2,\dots N$ measured at a fixed horizontal spacing Δd, its linear trend is first removed as with the program SPECTRA:

$$\bar{Z}(i) = Z(i) - a_0 - a_1 i, \quad i = 1,2,\dots N. \tag{6}$$

From the residuals $\bar{Z}(i)$, the variances of height differences at varying lags k are computed:

$$V(k) = \frac{1}{N-k} \sum_{i=1}^{N-k} \{\bar{Z}(i) - \bar{Z}(i+k)\}^2, \quad k = 1,2,\dots N/2 \tag{7}$$

Figure 1 represents the plot of natural logarithms of $V(k)$ versus k, also known as the log-log plot of variogram modelled as $V(k) = ck^{\beta}$. Based on computations with real data, the points tend to lose correlation at larger values of k (see points 10 to 16). Based upon the first few points (1 to 9) that are well correlated, a straight line may be drawn, from which slope β and the ordinate-intercept $\ln(c)$ are determined graphically.

β and $\ln(c)$ may also be determined analytically. Starting with the first three points $\ln(V(k))$, $k = 1,2,3$, a straight line function is determined by least squares. Residuals are computed and compared with a threshold that is equivalent to an acceptable graphical error. If residuals are less than the given threshold, then the process is repeated with the next point included, until at least one residual exceeds the given threshold. This is the way β and $\ln(c)$ are determined by this program. Hence, manual plotting is completely avoided.

The optimum sampling interval is

$$\Delta d_{opt} = L\Delta d \tag{8}$$

where L is computed from

$$\frac{\sigma^2_{int}}{c} = (L^{\beta})\left(\frac{2-2^{\beta-1}}{2^{\beta+1}}\right) \tag{9}$$

Program LINEAR

This program is based on the assumption that the DEM data are sufficient in density such that heights can be interpolated accurately by a straight line or planar function.

Given a sample profile $Z(i)$, $i = 1,2,\ldots N$ measured at a fixed horizontal spacing Δd, the optimum sampling interval Δd_{opt} is determined iteratively. Starting with a control spacing of $2\Delta d$, intermediate sample points are interpolated linearly and elevations compared with their respective measured elevations. The root mean square (RMS) value of the discrepancies is computed and compared with σ_{int}. If RMS is less than σ_{int}, the process is repeated by increasing the control spacing by Δd. If $k\Delta d$ is the control spacing for each iteration, $k = 2,3,\ldots K$ where K is the least value of k whereby σ_{int} is exceeded, then

$$\Delta d_{opt} = (K-1 + \varepsilon)\Delta d \tag{10}$$

where

$$\varepsilon = \frac{\sigma_{int} - RMS(K-1)}{RMS(K) - RMS(K-1)} \tag{11}$$

When $K = 2$, $\Delta d_{opt} = \Delta d$, assuming $RMS(1) = \sigma_{int}$.

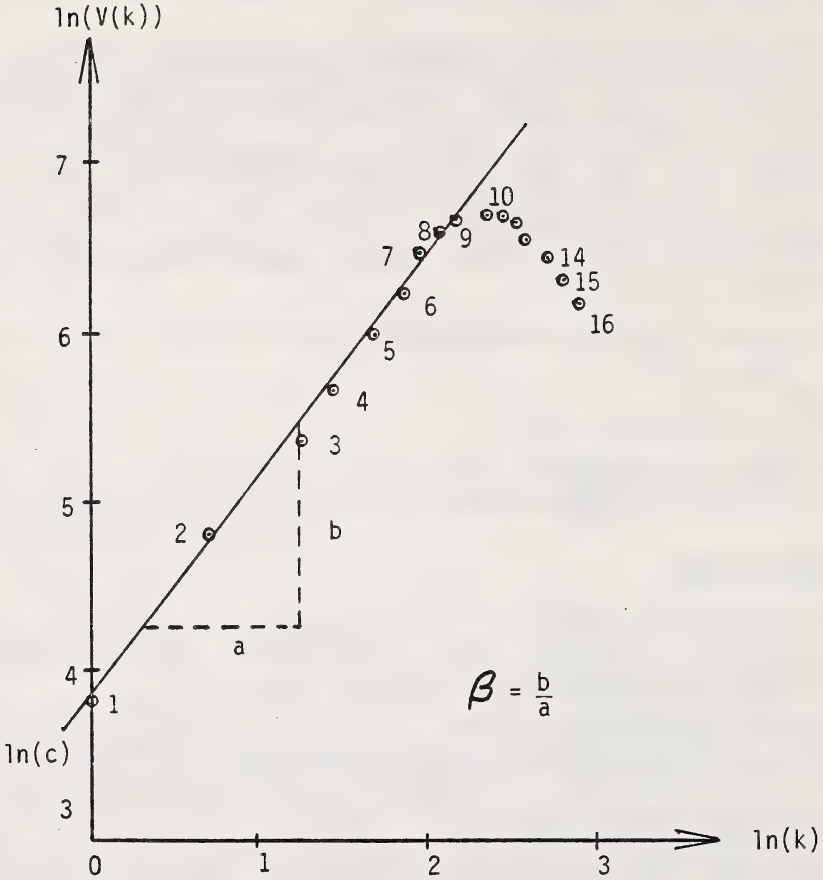

Fig. 1. LOG-LOG PLOT OF VARIOGRAM

Program RF

This program was developed primarily for analyzing terrain undulation based on the following parameters (Frederiksen, Jacobi and Kubik, 1984):

a) Relief is the height difference between the lowest and highest elevations of a given profile.
b) Slope is the first derivative of the profile at a point.
c) Wavelength is the average distance between successive points of zero slope (e.g. two hilltops).

In this program:

a) Relief is the average height difference Δz_{ave} between successive significant breakpoints.
b) Wavelength is the average distance, Δd_{ave} between successive significant breakpoints. The optimum sampling interval is

$$\Delta d_{opt} = (\Delta d_{ave})/2 \qquad (12)$$

c) Mean slope = $(\Delta z_{ave}/\Delta d_{ave})$ 100% (13)

Significant breakpoints are determined using the following criteria:

a) Height difference between any successive points exceeds σ_{int}.
b) If a point is linearly interpolated between its two neighboring points, and its computed elevation is compared with its sampled elevation, the discrepancy exceeds σ_{int}.

The roughness factor, R.F. in this paper is defined to be the mean slope (Equation 13) which will be used for analyzing the results of test comparisons in this paper.

TEST DATA

The programs were tested with real data obtained from wide angle photography, at a scale of 1:60 000 covering four contiguous stereomodels: 4200, 4202, 4204 and 4206, which had been used for compiling 1:20 000 provincial topographic series of the area with 10-metre contour lines (see Figure 2). The profiles were sampled along the north-south boundaries of the models at a fixed horizontal spacing of 25 m using an analytical stereoplotter (see Table 1). Profile 200 is situated on the edge of the Rocky Mountains. All the others are on moderately rolling or flat terrain.

The programs were tested and compared using the following sets of profiles:

a) Long profiles at fixed sample spacing (see Table 1).
b) Short profiles at fixed sample spacing.
c) Long profiles of varying sample spacings.

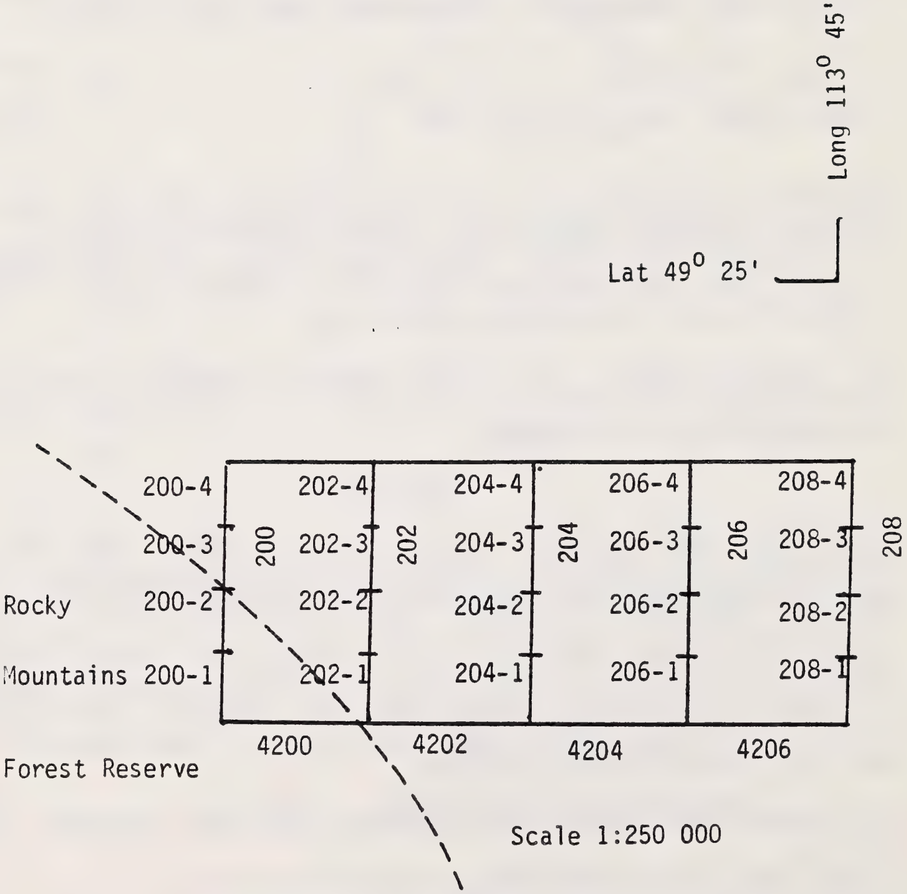

Fig. 2 TEST AREA

Short profiles were derived from the long ones by subdividing each profile into four parts (see Table 2). Profiles 200-1 and -2 (south half of profile 200) are on the rugged terrain of the Rocky Mountains as indicated by their roughness factors.

Profiles of varying sample spacings were derived from the long profiles to produce profiles with sample spacings of 50, 75 and 100 m. The original long profiles are also included (see Table 3). The roughness factors (R.F.) of derived profiles from each original profile seem to be invariant as shown in Table 3.

PROCEDURE

All sample profiles (Tables 1 to 3) were processed using all four programs: SPECTRA, LOGKV, LINEAR and RF. Input data to each program include:

a) Profile header/identifier
b) Required accuracy for interpolation σ_{int} (m)
c) Horizontal spacing Δd (m)
d) Profile data X, Y, Z (m)

The required accuracy for interpolation σ_{int} has been determined as follows (see the Appendix):

$$\sigma_{int} \approx (\sigma^2_{spec} - \sigma^2_{AT} - \sigma^2_{setup} - \sigma^2_{samp})^{1/2} \tag{14}$$

$$\sigma_{spec} \approx \frac{C.I.}{2(1.64)} \approx \frac{10}{2(1.64)} \approx 3.05 \text{ m} \tag{15}$$

$$\sigma_{samp} \approx \sigma_{setup} \approx \frac{H}{(C\text{-factor})(1.64)(2)} \tag{16}$$

$$\approx \frac{9180}{2500\ (1.64)2} \approx 1.12 \text{ m}$$

Photo scale is 1:60 000. An analytical stereoplotter is assumed to be used for sampling.

$$\sigma_{AT} \approx (RMS^2_{CP} + RMS^2_{TP})^{1/2} \approx (20^2 + 15^2)^{1/2} = 25\mu\text{m}, \tag{17}$$

at image scale or 1.50 m in ground scale.

$$\sigma_{int} \approx (3.05^2 - 1.50^2 - 1.12^2 - 1.12^2)^{1/2} = 2.13 \text{ m}$$

Table 1. Sample Profiles

Profile	Total No. of Points	Length (m)	R.F. (%)
200	438	10 295	18.7
202	479	11 950	6.9
204	457	11 400	4.6
206	427	10 650	4.0
208	439	10 950	4.2

Table 2. Short Profiles

Profile	Total No. of Points	Length (m)	R.F. (%)
200-1	112	2775	36.0
-2	114	2825	24.7
-3	112	2775	8.1
-4	100	2475	3.6
202-1	127	3150	4.5
-2	114	2825	7.8
-3	114	2825	6.0
-4	124	3075	9.5
204-1	101	2500	6.0
-2	114	2825	5.1
-3	114	2825	3.2
-4	128	3175	4.3
206-1	97	2400	2.1
-2	114	2825	5.7
-3	114	2825	4.1
-4	102	2525	4.0
208-1	123	3050	5.1
-2	114	2825	5.1
-3	104	2575	4.2
-4	98	2425	3.6

Table 3. Profiles of Varying Sample Spacings

Profile	Sample Spacing(m)	Total No. of Points	Length (m)	R.F. (%)
200	25	438	10 925	18.7
	50	219	10 900	18.2
	75	146	10 875	18.4
	100	110	10 900	17.0
202	25	479	11 950	6.9
	50	240	11 950	6.7
	75	160	11 925	6.6
	100	120	11 900	6.6
204	25	457	11 400	4.6
	50	229	11 400	4.6
	75	153	11 400	4.3
	100	115	11 400	4.2
206	25	427	10 650	4.0
	50	214	10 650	4.0
	75	143	10 650	4.0
	100	107	10 600	3.9
208	25	439	10 950	4.2
	50	220	10 950	4.1
	75	147	10 950	4.1
	100	110	10 900	3.9

RESULTS

Long Profiles

The profiles have been rearranged in decreasing order of R.F.'s (see Table 4). It can be noted that:

a) The mean sampling intervals (14) are increasing with decreasing R.F.'s.
b) Differences from the mean sampling interval (3), (6), (9), (12) are within ±18 m for profile of highest R.F. (200). Differences are inconsistent with the other profiles.
c) RMS's of percent differences from the mean are all less than 37%.
d) LOGKV has the least RMS while SPECTRA has the largest.
e) RF computes the most conservative sampling intervals, while SPECTRA and LINEAR tend to be the most optimistic. (See mean percent differences).
f) LOGKV has the most favourable percent differences (7) with a maximum of 22% only in absolute value. On the other hand, SPECTRA has the worst percent differences (4) with a maximum of 54%.

Short Profiles

The profiles have been rearranged in decreasing order of R.F.'s (see Table 5). It can be noted that:

a) The mean sampling intervals (14) have an increasing trend with decreasing R.F.'s, except for some inconsistencies at R.F. <7.8.
b) Differences from the mean sampling intervals (3), (6), (9), (12) are within ±4 m for profiles of higher R.F. (200-1 and -2). Differences are inconsistent for the other profiles.
c) RMS's of percent differences from the mean intervals are all less than 34%.
d) LOGKV has the least RMS, while RF has the largest.
e) RF computes the most conservative sampling intervals, while SPECTRA tends to be the most optimistic. (See mean of percent differences).
f) LOGKV has the most favourable percent differences (7) with a maximum of 23%. On the other hand, SPECTRA has the worst percent differences (4) with a maximum of 60%.

Profiles of Varying Sample Spacings

Computed sampling intervals of 50, 75 and 100 m profiles were compared with those of 25 m profiles as computed by the programs (see Table 6). It can be noted that:

(a) Differences (3), (6), (9) in absolute values are generally increasing with increasing sampling intervals for all programs.
(b) Except for LINEAR, maximum percent differences (4), (7), (10) in absolute values are on the roughest profile (200).
(c) LOGKV has the least RMS, while SPECTRA has the largest.
(d) LOGKV is the most conservative, while SPECTRA is the most optimistic. (See means of percent differences).

Table 4 Results With Long Profiles

Profile	SPECTRA			LOGKV			LINEAR			RF			Mean	R.F.
	Int(m)	Diff	%	Int	Diff	%	Int(m)	Diff	%	Int(m)	Diff	%	(m)	(%)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
200	29	-13	-31	41	- 1	- 2	60	18	43	38	- 4	-10	42	18.7
202	123	23	23	88	-12	-12	111	11	11	77	-23	-23	100	6.9
204	200	64	47	116	-21	-15	142	6	4	88	-49	-36	137	4.6
208	139	- 6	- 4	147	2	1	178	33	23	116	-29	-20	145	4.2
206	229	81	54	116	-32	-22	159	11	7	89	-59	-40	148	4.0
RMS			36			13			23			28		
Mean			18			-10			18			-26		

Notes:
(1) Profile identifier

(2), (5), (8), (11) Computed sampling intervals

(14) Mean sampling interval

(3), (6), (9), (12) Differences from the mean

(4), (7), (10), (13) Percent differences with respect to mean

(15) Roughness factor

Table 5 Results With Short Profiles

	SPECTRA			LOGKV			LINEAR			RF			Mean	R.F.
Profile	Int (m)	Diff	%	Int (m)	Diff	%	Int (m)	Diff	%	Int (m)	Diff	%	(m)	(%)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
200-1	29	4	17	24	- 1	- 3	25	0	1	21	- 4	-15	25	36.0
200-2	31	2	6	33	4	13	25	-4	-15	28	- 1	- 4	29	24.7
202-4	48	-33	-41	100	19	23	117	36	44	59	-22	-27	81	9.5
200-3	200	69	52	103	-28	-22	154	23	17	68	-63	-48	131	8.1
202-2	94	19	25	59	-16	-21	86	11	15	61	-14	-19	75	7.8
204-1	133	36	37	91	- 6	- 6	92	- 5	- 5	73	-24	-25	97	6.0
202-3	229	86	60	120	-24	-16	137	- 7	- 5	88	-56	-39	144	6.0
206-2	114	21	23	82	-11	-12	108	15	16	68	-25	-27	93	5.7
208-2	123	9	8	109	- 5	- 4	115	1	1	109	- 5	- 4	114	5.1
204-2	178	60	51	94	-24	-20	129	11	9	71	-47	-40	118	5.1
208-1	145	10	7	146	11	8	180	45	33	71	-65	-48	136	5.1
202-1	123	10	9	102	-11	-10	115	2	2	111	- 2	- 2	113	4.5
204-4	229	64	39	154	-11	- 7	183	18	11	93	-72	-44	165	4.3
208-3	229	14	6	215	0	0	212	- 3	- 2	205	-10	- 5	215	4.2
206-3	178	38	27	137	- 4	- 2	146	6	4	101	-40	-28	141	4.1
206-4	200	44	28	138	-18	-11	206	50	32	79	-77	-49	156	4.0
208-4	229	30	15	197	- 2	- 1	234	35	18	135	-64	-32	199	3.6
200-4	267	66	33	203	2	1	218	17	8	118	-84	-41	202	3.6
204-3	145	-42	-22	202	15	8	283	96	51	118	-69	-37	187	3.2
206-1	320	78	32	260	18	7	290	48	20	100	-143	-59	243	2.1
RMS			31			12			21			34		
Mean			21			- 4			13			-30		

Notes:

(1) Profile identifier
(2), (5), (8), (11) Computed sampling intervals
(14) Mean sampling interval
(3), (6), (9), (12) Differences from the mean
(4), (7), (10), (13) Percent differences with respect to mean
(15) Roughness factor

Table 6 Results With Profiles of Varying Sample Spacings

	50 m			75 m			100 m			25 m
Profile	Int (m)	Diff	%	Int (m)	Diff	%	Int (m)	Diff	%	Int (m)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
SPECTRA										
200	53	24	83	76	47	162	100	71	245	29
202	126	3	2	145	22	18	152	29	24	123
204	206	6	3	192	- 8	- 4	229	29	15	200
206	200	-29	-13	223	- 6	- 3	237	8	3	229
208	139	0	0	150	11	8	168	29	21	139
RMS = 79.5 Mean = 37.6										
LOGKV										
200	29	-12	-29	21	-20	-49	20	-21	-51	41
202	75	-13	-15	66	-22	-25	53	-35	-40	88
204	108	- 8	- 7	91	-25	-22	100	-16	-14	116
206	104	-12	-10	92	-24	-21	94	-22	-19	116
208	128	-19	-13	115	-32	-22	116	-31	-21	147
RMS = 27.0 Mean = -23.9										
LINEAR										
200	50	-10	-17	75	15	25	100	40	67	60
202	50	-61	-55	75	-36	-32	100	-11	-10	111
204	116	-26	-18	75	-67	-47	100	-42	-30	142
206	133	-26	-16	75	-84	-53	100	-59	-37	159
208	156	-22	-12	156	-22	-12	100	-78	-44	178
RMS = 36.2 Mean = -19.4										
RF										
200	53	15	39	57	19	50	74	36	95	38
202	83	6	8	82	5	6	88	11	14	77
204	91	3	3	114	26	30	127	39	44	88
206	106	17	19	113	24	27	121	32	36	89
208	112	- 4	- 3	112	- 4	- 3	133	17	15	116
RMS = 35.4 Mean = 25.3										

Notes:
(1) Profile identifier
(2), (5), (8), (11) Computed sampling intervals
(3), (6), (9) Differences of (2), (5), (8) from (11)
(4), (7), (10) Percent differences of (3), (6), (9) with respect to (11)
RMS's and means are of those percent differences (4), (7), (10).

(e) LOGKV has the most favourable percent differences (7) with a maximum of 51%. On the other hand, SPECTRA has the worst percent differences (7) with a maximum of 245%.

OBSERVATIONS

From the results, we can conclude the following:

(a) Except in flat terrain (R.F. ≤8.1), sampling intervals computed by all programs are increasing with decreasing roughness factors. Hence, the formulation of roughness factor in this paper is indeed consistent.

(b) In rough terrain (R.F. ≥18.7), all programs agree on the computed sampling intervals within ±18 m of the mean with short and long profiles. Hence, any of the programs appears suitable for rough terrain, regardless of the profile length.

(c) RMS's of percent differences with long and short profiles are within 37%, for all programs. Hence, any of the programs is potentially suitable for the purpose of determining the optimum sampling interval.

(d) LOGKV appears to be the most suitable program to use with any terrain roughness or profile length.

(e) With long and short profiles, RF tends to compute the most conservative sampling interval, while SPECTRA tends to be the most optimistic. LINEAR tends to be the most optimistic also with long profiles.

(f) Sampling intervals computed by any of these programs, become less reliable with increasing sample spacings and increasing roughness of the terrain.

(g) LOGKV appears to be the most suitable with profiles of sparse sample points.

VALIDATION OF OBSERVATIONS

Procedure

For the purpose of validating the observations made previously, model 4200 (see Figure 2) was sampled three times, at grid intervals of 75 and 130 m, for the entire model, and 25 m for its southwest quarter portion. Breaklines, hilltops and depressions were also measured. From each grid sampling combined with breaklines, hilltops and depressions, contour lines at 10 m interval were interpolated and plotted at 1:20 000 scale. Each contour plot was tested for accuracy by spot heighting intersections of a drawn profile with index contour lines on an analytical stereoplotter.

Results and Discussion

The accuracy of the discrepancies of contour lines may be estimated as follows:

$$\text{Disc} = \text{CE} - \text{ME} \tag{18}$$

where

Disc = discrepancy
CE = contour elevation
ME = measured elevation

By the law of propagation of variances and disregarding correlations

$$\sigma^2_{\text{Disc}} \approx \sigma^2_{\text{CE}} + \sigma^2_{\text{ME}} \tag{19}$$

$$\sigma^2_{\text{CE}} \approx \sigma^2_{\text{spec}}$$

$$\sigma^2_{\text{ME}} \approx \sigma^2_{\text{AT}} + \sigma^2_{\text{setup}} + \sigma^2_{\text{samp}} \tag{20}$$

Therefore,

$$\sigma^2_{\text{Disc}} \approx \sigma^2_{\text{spec}} + \sigma^2_{\text{AT}} + \sigma^2_{\text{setup}} + \sigma^2_{\text{samp}} \tag{21}$$

$$\sigma_{\text{Disc}} \approx (3.05^2 + 1.50^2 + 1.12^2 + 1.12^2)^{1/2} \approx 3.8 \text{ m}$$

From the discrepancies of the contour elevations, it is evident that the contour elevations of the contour plot from the 130 m grid sampling do not meet the specified accuracy (see Table 7). The discrepancies from the 75 m grid partially meet the specified accuracy (see points 20 to 36 with RMS of 2.9 m). Points 1 to 19 are on the rugged terrain of the Rocky Mountains. Those from the 25 m grid meet the specified accuracy with RMS of 3.2 m.

From the results, we can conclude that the grid interval of 130 m for the entire model and 75 m for its rugged portion do not meet the specified accuracy. On the other hand, 75 m grid interval for the rolling and flat terrain portion, and 25 m for the rugged portion, both meet the specified accuracy. Based on this conclusion, we can identify sample profiles that are representative of the model's terrain characteristics based on their locations and computed sampling intervals by the four programs. The sample profiles should be located on the portion of the model represented by the 75 or 25 m grid intervals, and that the range of sampling intervals computed for each profile includes 75 or 25 m as the case may be. After identifying the relevant profiles, discrepancies of computed sampling intervals from 75 or 25 m have been determined and results summarized in Tables 8 and 9.

Table 7 Discrepancies of Contour Lines

Point Number	Discrepancies (m)		
	130 m	75 m	25 m
1	-30	0	-2
2	23	23	-3
3	70	28	-3
4	70	24	-5
5	6	21	0
6	-51	5	2
7	-59	-5	-3
8	39	-9	-2
9	81	33	-2
10	80	33	-4
11	9	25	-1
12	-48	9	-1
13	-52	3	-3
14	-54	- 4	-10
15	-58	- 2	2
16	11	-10	6
17	49	15	5
18	57	1	-2
19	154	21	2
20	103	5	1
21	-	0	3
22	56	2	6
23	57	4	4
24	51	2	-1
25	51	3	4
26	- 2	2	2
27	-48	- 4	4
28	- 1	0	2
29	46	- 1	3
30	0	- 3	-1
31	-52	- 1	0
32	6	- 2	-1
33	51	5	1
34	2	2	0
35	5	3	0
36	-	3	-1
37	-	-	-1

Table 8 Differences of Sampling Intervals from 75 m

	R.F.	SPECTRA			LOGKV			LINEAR			RF		
Profile	(%)	Int (m)	Diff	%	Int (m)	Diff	%	Int (m)	Diff	%	Int (m)	Diff	%
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
202-4	9.5	48	-27	-36	100	25	33	117	42	56	59	-16	-21
200-3	8.0	200	125	167	103	28	37	154	79	105	68	- 7	- 9
202-2	7.8	94	19	25	59	-16	-21	86	11	15	61	-14	-19
Mean				52			16			59			-16
200 (75)	18.4	76	1	1	21	-54	-72	75	0	0	57	-18	-24
(100)	17.0	100	25	33	20	-55	-73	100	25	33	74	- 1	- 1
202 (50)	6.7	126	51	68	75	0	0	50	-25	-33	83	8	11
(75)	6.6	145	70	93	66	- 9	12	75	0	0	82	7	9
(100)	6.6	152	77	103	53	-22	-29	100	25	33	88	13	17
Mean				60			-32			7			2

Notes:

(1) Profile identifier
(2) Roughness factor
(3), (6), (9), (12) Computed sampling intervals
(4), (7), (10), (13) Differences from 75 m
(5), (8), (11), (14) Percent differences with respect to 75 m

Table 9 Differences of Sampling Intervals from 25 m

Profile	R.F. (%)	SPECTRA Int (m)	SPECTRA Diff	SPECTRA %	LOGKV Int (m)	LOGKV Diff	LOGKV %	LINEAR Int (m)	LINEAR Diff	LINEAR %	RF Int (m)	RF Diff	RF %
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
200-1	36.0	29	4	16	24	- 1	- 4	25	0	0	21	- 4	-16
200-2	24.7	31	6	24	33	8	32	25	0	0	28	3	12
Mean				20			14			0			- 2

Notes:

(1) Profile identifier
(2) Roughness factor
(3), (6), (9), (12) Computed sampling intervals
(4), (7), (10), (13) Differences from 25 m
(5), (8), (11), (14) Percent differences with respect to 25 m

None of the long profiles (Table 4) were found to be representative of the regions represented by 75 or 25 m grid intervals. From the short profiles in Table 5, three were found to be representative of the 75 m grid region and two of the 25 m grid region. From profiles of varying sample spacings (Table 6), five of them were found to be representative of the 75 m grid region, while none of them were of the 25 m grid region.

In rough terrain (R.F. ⩾24.7), the computed sampling intervals are within ±8 m compared with 25 m (see Table 9). LOGKV or RF appears to be the most suitable program to use with short profiles of any terrain roughness (see Tables 8 & 9). RF tends to be the most conservative while LINEAR tends to be the most optimistic (see Table 8). RF appears to be the most suitable program with profiles of sparse sample points.

RECOMMENDATIONS

Guidelines for Using the Programs

Based on previous analyses the most suitable program(s) will be recommended for each particular case. With progressive sampling, program SPECTRA is the most suitable for determining the zero-order sampling interval because it is the most optimistic of all the programs.

Without progressive sampling, one has to be more conservative. Table 10 provides some guidelines as to the most suitable programs to use depending on the following profile attributes:

(a) Roughness of the terrain
- rough terrain (R.F. ⩾15)
- flat terrain (R.F. <15)

(b) Length of sample profile
- long (length ⩾1.75 model base)
- short (length <1.75 model base)

(c) Profile sampling rate
- high ($\Delta d \leqslant 0.8$ mm at image scale)
- low ($\Delta d > 0.8$ mm at image scale)

The recommended programs are based on means of percent differences of computed sampling intervals obtained from the tables referenced and profiles used. A program is recommended if percent differences are within ±50%. If more than one program is recommended, the programs are arranged in the order of increasing degree of optimism. In other words, the first program is the most conservative and the last is the most optimistic. Cases (c) and (g) should be avoided, because sample points may be too few to produce reliable results. If results of validation (Tables 8 and 9) do not agree with those of previous comparisons (Tables 4, 5 and 6), then the former prevail.

Table 10. Program Guidelines

Profile Attributes	Most Suitable Programs	Table(s) Referenced	Profile(s) Used
a) Rough, long, high	LOGKV, LINEAR or RF	Table 4 Table 6	200 200 (50 m)
b) Rough, long, low	RF, LINEAR or SPECTRA	Table 8	200 (75m, 100m)
c) Rough, short, low	-	-	-
d) Rough, short, high	RF, LINEAR, LOGKV or SPECTRA	Tables 5 & 9	200-1 & -2
e) Flat, long, high	LINEAR, LOGKV or RF	Table 8	202 (50 m)
f) Flat, long, low	LOGKV, RF or LINEAR	Table 8	202 (75, 100 m)
g) Flat, short low	-	-	-
h) Flat, short high	RF or LOGKV	Table 8	200-3, 202-2 & -4

Note: No data was available for (c) and (g).

Recommended Sampling Strategy.

The following sampling strategy is recommended if progressive sampling is not available:

a) Measure breaklines, hilltops and depressions, preferably plotted on the manuscript or shown on an interactive graphics display interfaced with the stereoplotter.
b) Decide whether or not the model will be partitioned into regions of varying grid sizes, depending on the variations of terrain undulation.
c) If the decision is made to partition the model, then delineate boundaries of regions and excluded areas (water bodies, cloud cover, etc.), preferably indicated on a manuscript or shown on an interactive graphics display.
d) Measure at least one sample profile at a fixed sample spacing, in each region or the entire model. Each profile should be representative of the terrain undulation. Avoid breaklines that have been measured previously. Each profile should be as long and as dense as possible for more reliable results.
e) Compute σ_{int} as in the Appendix.
f) Run program RF to determine the roughness factor for each profile.
g) From Table 10, use the most appropriate program, given the profile attributes.
h) Depending on the results, decide whether to use one or two sizes for grid sampling in the two perpendicular directions for each region or entire model. If only one size is used, then use the mean sampling interval.
i) Sample each region or the entire model using the final sampling interval(s), in stationary mode.
j) Generate contour lines for the entire model and plot them at the same scale as the manuscript on the plotting table of the stereoplotter or display them on an interactive graphics display.
k) Scan the model and check the contour lines for completeness, fidelity and accuracy. It may be advantageous to project the contour lines on the optical system of the stereoplotter for simultaneous viewing of the model and contour lines. However, continuous display of contour lines might interfere with stereoscopic perception depending on the density of the contour lines. Hence, it may be useful to have a facility for regulating the light intensity of contour lines at the operators own discretion.
l) While scanning the model, additional points may be sampled as required and contour lines regenerated. An example of an interactive graphics DEM sampling is presented in Ostman(1982).

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REFERENCES

Ayeni, O.O., 1982. "Optimum Sampling for Digital Terrain Models: A Trend Towards Automation", Photogrammetric Engineering & Remote Sensing, Vol. XLVIII, No. 11, pp. 1687-1694

Frederiksen, P., O. Jacobi and K. Kubik, 1983. "Measuring Terrain Roughness by Topological Dimension", ISPRS Working Group III/3 Colloquium, Stockholm, Sweden, April 18-20, 1983

Frederiksen, P., O. Jacobi and K. Kubik, 1984. "Modelling and Classifying Terrain", ISPRS Congress, Commission III, Rio de Janeiro, Brazil, July 1984

Fritsch, D. 1984. "Proposal for the Determination of the Least Sampling Interval for DEM Data Acquisition", Chair of Photogrammetry, Technical University of Munich, Federal Republic of Germany

Fritsch, D. 1985. "Some Remarks on Duality Between Fourier Series and Discrete Fourier Transform", Allegemeine Vermesungnachrichten International Supplement

Makarovic, B. 1977. "Composite Sampling for Digital Terrain Models", ITC Journal, 1977-3, pp. 406-433

Ostman, A. 1982. "Design of Interactive Graphics for DEM Sampling", ISPRS Commission III Symposium, Helsinki, Finland, June 7-11, 1982

Thorpe, J.A. 1984. "CPS: Computed Photo Scale", Photogrammetric Engineering & Remote Sensing, Vol. L, No. 11, p. 1567

APPENDIX

DETERMINATION OF REQUIRED ACCURACY FOR INTERPOLATION

The accuracy of contour lines interpolated from DEM data is influenced by the following factors:

a) Aerial triangulation
b) Model setup
c) Sampling process
d) Interpolation

Aerial triangulation accuracy σ_{AT} may be estimated as follows:

$$\sigma_{AT}^2 \approx RMS^2{}_{CP} + RMS^2{}_{TP} \tag{22}$$

where

RMS_{CP} ≈ RMS of vertical control point residuals after adjustment

RMS_{TP} ≈ RMS of vertical tie point residuals after adjustment

If no aerial triangulation has been done, such as in case of full model control or levelling points transferred from other triangulated photography, then use the appropriate standard deviation for those points.

Model setup accuracy σ_{setup} is estimated as follows:

$$\sigma_{setup} \approx \frac{C.I.}{(2)(1.64)} \tag{23}$$

$$\text{C-factor} \approx \frac{H}{C.I.} \tag{24}$$

where

C.I. = least contour interval possible with a given combination of photography, instrumentation and operator.
1.64 = χ^2 distribution factor, one dimension at 90% confidence interval
H = flying height above mean ground elevation.

Combining the above two equations (23) and (24),

$$\sigma_{setup} \approx \frac{H}{(\text{C-factor})(2)(1.64)} \tag{25}$$

For C-factors, see Table 11. The sampling accuracy, σ_{samp} may be estimated to be in the same order as σ_{setup}. Accuracy of interpolating contour lines σ_{int} is influenced mainly by spacing and accuracy of sample points, and interpolation algorithm.

All of the above error factors when combined vectorially should not exceed the required error of contour lines,

$$\sigma_{spec} \approx \frac{C.I.}{(2)(1.64)} \tag{26}$$

Hence, the error budget is:

$$\sigma^2_{spec} \approx \sigma^2_{AT} + \sigma^2_{setup} + \sigma^2_{samp} + \sigma^2_{int} \tag{27}$$

Solving for σ_{int},

$$\sigma_{int} \approx (\sigma^2_{spec} - \sigma^2_{AT} - \sigma^2_{setup} - \sigma^2_{samp})^{1/2} \tag{28}$$

Table 11 C-Factors (Thorpe, 1984)

Instrument	C-Factors Commercial	C-Factors U.S. Federal Agencies
All analytical plotters	3000	2500
Zeiss Planimat	2400	2100
Wild A10	2400	2100
Wild Aviomap	2400	2100
Santoni IV	2400	2100
Kern PG3	2400	2100
Jena Stereometrograph	2400	2100
Wild AG1	2000	1800
Wild A8	2000	1800
Santoni III	2000	1800
Santoni IIC	2000	1800
Galileo G7	2000	1800
Galileo G6	2000	1800
Kern PG2	2000	1800
Zeiss Planitop	1800	1500
Wild B8	1800	1500
Jena Topocart	1800	1500
Kelsh	1500	1200

APPENDIX 2

DETERMINATION OF OPTIMUM SAMPLING INTERVAL IN GRID SAMPLING OF DIGITAL ELEVATION MODELS FOR LARGE-SCALE APPLICATION

(Presented Paper, ISPRS Commission III Symposium, August 19-22, 1986, Rovaniemi, Finland)

DETERMINATION OF OPTIMUM SAMPLING INTERVAL IN GRID SAMPLING OF DIGITAL ELEVATION MODELS FOR LARGE-SCALE APPLICATION

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ACKNOWLEDGEMENTS

REFERENCES

DETERMINATION OF OPTIMUM SAMPLING INTERVAL IN GRID SAMPLING OF DIGITAL ELEVATION MODELS FOR LARGE-SCALE APPLICATION

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ABSTRACT

Four computer programs for determining the optimum sampling interval in grid sampling of digital elevation models (DEM) by photogrammetric methods, were tested with real data using large-scale photography and results compared. These programs were SPECTRA, LOGKV, LINEAR and RF. They were tested and compared using sample profiles obtained using an analytical stereoplotter from large-scale photography: 1:4 000 with flat terrain, and 1:8 000 with rough terrain. For the purpose of having absolute comparisons, one stereomodel from each photo scale was sampled completely up to three times at various grid sampling intervals determined by the programs. Together with the measurements of breaklines, hilltops and depressions, contour lines were interpolated and plotted from each grid sampling. Contour lines of each plot were then tested for accuracy using an analytical stereoplotter. From the results, guidelines for using the programs were formulated and are recommended.

1. BACKGROUND

When Alberta Energy and Natural Resources (now Alberta Forestry) embarked on acquisition of digital elevation models required basically for the new provincial topographic series at a scale of 1:20 000 with 10 m contour interval in early 1984, the department and the private mapping companies were both uncertain as to the optimum grid sampling interval. Details of this project are presented in /3/.

In an effort to learn from experts regarding DEM data acquisition, processing, databanking and applications, ABSM held a DEM workshop in Edmonton on October 10-12, 1984 /2/. The workshop was organized and chaired by Mr. M.A.G. Toomey, Mapping Branch, ABSM. It was attended by DEM experts from Canada, United States of America, Federal Republic of Germany, Austria and Switzerland, in addition to local mapping contractors and departmental staff engaged in DEM.

After the workshop, one of the participants Dr. D. Fritsch, Technical University of Munich, F.R.G. sent us the source code listing of program SPECTRA which is based on Fourier transformation. About the same time, Dr. K. Kubik, The Ohio State University, U.S.A stopped over in Edmonton from Denmark while on his way to Australia. In two days he introduced and programmed LOGKV which is based on self-similarity concept. Both programs SPECTRA and LOGKV were tested and compared using real data. However, results appeared to be inconsistent. Consequently, a new program was developed called LINEAR, by the author. To determine the roughness factors of the profiles, used as the basis for comparisons, program RF was developed, also by the author. Incidentally, the program was found to be capable of computing optimum sampling interval also. Programs LINEAR and RF are based on linear interpolation. For details about the four programs, see /1/.

All of these programs were then tested with real data using 1:60 000 wide-angle photography used for the 1:20 000 provincial topographic series. Results were compared relatively and absolutely from which guidelines for using the programs and a sampling strategy were formulated /1/.

These guidelines and strategy are summarized as follows:

a) Data acquisition of one or more sample profiles depending on the model's roughness.
b) Estimation of the error budget for interpolating heights from the DEM data to be acquired.
c) Running of program RF, given the sample profile(s) and error budget, to determine the roughness factor of each profile.
d) Selection of the most suitable program to use, knowing the roughness factor, length and sample spacing of each profile.
e) Running the selected program(s), given the sample profile(s) and error budget, to determine the optimum sampling interval(s).

To test the feasibility of using these programs with large-scale photography, a similar experiment was undertaken. This paper deals with data, procedures and results of such an experiment.

2. TEST DATA

2.1 Raymond (1:4 000)

This data was obtained from the photogrammetric control coverage for the town of Raymond located in southern Alberta, 30 miles north of the Alberta - Montana, U.S.A border. Sample profiles were measured along the north-south boundaries and bases of models: 2152, 2153 and 2154 (see Figure 1). Sample spacing was 3 m with the use of an analytical stereoplotter.

Profiles 152, 153, 154 and 155 were categorized as long profiles. Profiles 1521, 1531 and 1541 were categorized as short profiles, supplemented with subdivisions of the long ones: 152-1, 152-2, 153-1, 153-2, 154-1, 154-2, 155-1 and 155-2. Profiles of varying sample spacings were obtained by deriving new profiles of 6 and 9 m sample spacings from the long ones.

2.2 Crowsnest Pass (1:8 000)

To obtain rough terrain profiles, one stereomodel was chosen from the photogrammetric control coverage for the municipality of Crowsnest Pass located near the Rocky Mountains, 50 miles north of the Alberta-Montana, U.S.A. border. Sample profiles were measured along the north-south edges and its base at 4 m sample spacing with an analytical stereoplotter (see Figure 2).

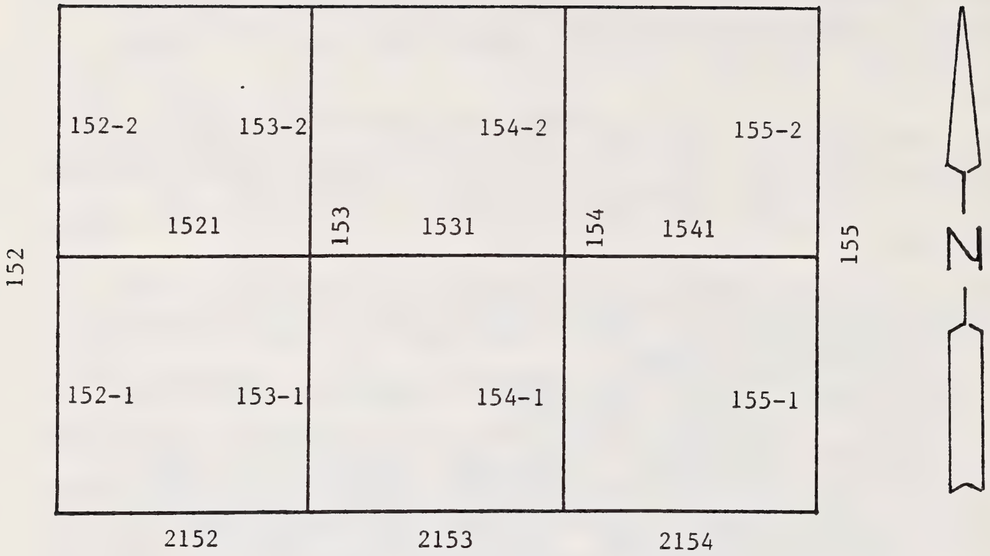

FIG. 1 RAYMOND DATA

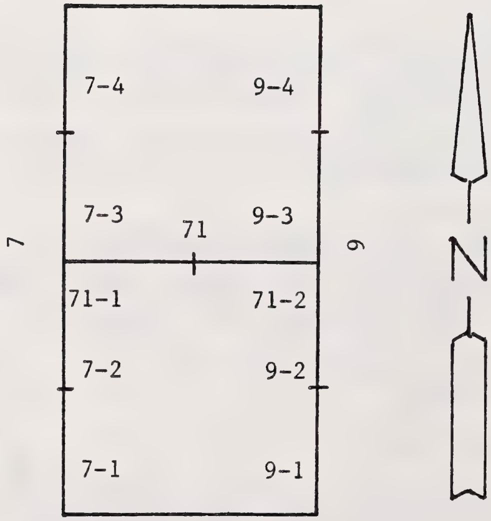

FIG. 2 CROWSNEST PASS DATA

Profiles 7 and 9 were categorized as long profiles. Profile 71 was categorized as short profile, supplemented with its subdivisions and those of the long ones: 71-1, 71-2, 7-1, 7-2, 7-3, 7-4, 9-1, 9-2, 9-3and 9-4. Profiles of varying sample spacings were derived from the long ones to produce new profiles at 8, 12, 16 and 20 m sample spacings.

3. PROCEDURE

All sample profiles were processed using all four programs: SPECTRA, LOGKV, LINEAR and RF. Input data to each program included:

a) Profile header/identifier
b) Error budget σ_{int}(m)
c) Sample spacing Δd (m)
d) Profile data X,Y,Z (m)

The error budget σ_{int} was determined as follows (see the Appendix, /1/):

$$\sigma_{int} \approx \left(\sigma^2_{spec} - \sigma^2_{AT} - \sigma^2_{setup} - \sigma^2_{samp}\right)^{1/2} \qquad (1)$$

For the Raymond profiles:

$$\sigma_{spec} \approx \frac{C.I.}{2(1.64)} \approx \frac{0.5}{2(1.64)} \approx 0.15m \qquad (2)$$

$\sigma_{samp} \approx \sigma_{setup} \approx 0.03$m, mean of RMS's of height residuals after exterior orientation of the three models measured.

$$\sigma_{AT} \approx \left(RMS^2_{TP} + RMS^2_{CP}\right)^{1/2} \qquad (3)$$

$$\approx \left(0.03^2 + 0.06^2\right)^{1/2} \approx 0.07 \text{ m}$$

$$\sigma_{int} \approx \left(0.15^2 - 0.07^2 - 0.03^2 - 0.03^2\right)^{1/2} \approx 0.12 \text{ m}$$

For the Crowsnest Pass profiles:

$$\sigma_{spec} \approx \frac{1}{2(1.64)} \approx 0.31 \text{ m}$$

$\sigma_{samp} \approx \sigma_{setup} \approx 0.05$ m, RMS of height residuals after exterior orientation of this model.

$$\sigma_{AT} \approx \left(0.06^2 + 0.11^2\right)^{1/2} \approx 0.12 \text{ m}$$

$$\sigma_{int} \approx \left(0.31^2 - 0.12^2 - 0.05^2 - 0.05^2\right)^{1/2} \approx 0.28m$$

4. RESULTS

4.1 Long profiles

These profiles are arranged in decreasing order of roughness factors (see Table 1). It can be noted that:

a) The mean sampling intervals (14) are increasing with decreasing roughness factors (15).

- b) Except for program SPECTRA, differences (6), (9) and (12) are increasing in absolute values with decreasing R.F.'s (15).
- c) Differences from the mean sampling interval (3), (6), (9) and (12) are within ± 3 m for profiles of higher R.F.'s (R.F. >2.3)
- d) RMS's of percent differences are all less than 33%.
- e) RF computed the most conservative sampling intervals, while LINEAR computed the most optimistic ones. See means of percent differences.
- f) Percent differences are within ± 50%.

4.2 Short profiles

These profiles are arranged in decreasing order of R.F.'s (see Table 2). It can be noted that:

- a) Differences from the mean sampling interval (3), (6), (9) and (12) are within ±2 m for profiles of higher R.F.'s (R.F.>11.3).
- b) RMS's of percent differences are all less than 35%.
- c) RF computed the most conservative sampling intervals, while LINEAR computed the most optimistic ones. See means of percent differences.
- d) LOGKV was the only program with percent differences (7) within ±42%

4.3 Profiles of varying sample spacings

4.3.1 Raymond profiles

Computed sampling intervals of 6 and 9 m profiles were compared with the mean sampling intervals of the original long profiles (see Table 3). It can be noted that:

- a) Only program SPECTRA has a definite increasing trend of percent differences with increasing sample spacings.
- b) RMS's of percent differences are all less than 43%.
- c) RF computed the most conservative sampling intervals, while SPECTRA computed the most optimistic ones. See means of percent differences.
- d) Only program SPECTRA has percent differences less than 33%.

4.3.2 Crowsnest Pass profiles

Computed sampling intervals of 8, 12, 16 and 20 m profiles were compared with the mean sampling intervals of the original long profiles (see Table 4). It can be noted that:

- a) All programs have increasing trends of percent differences with increasing sample spacings.
- b) Only program LOGKV has RMS less than 40%.
- c) LOGKV computed the most conservative sampling intervals, while SPECTRA computed the most optimistic ones. See means of percent differences.
- d) All programs have percent differences exceeding ±50%.

5. ABSOLUTE COMPARISON

5.1 Procedures

5.1.1 Raymond stereomodel

To validate the observations made previously, model 2153 (see figure 1) was sampled three times at grid intervals of 20, 51 and 62 m. Breaklines, hilltops and depressions were also measured. From each grid sampling combined with breaklines, hilltops and depressions, contour lines at 0.5 m interval were interpolated and plotted at 1:1 000 scale. Each contour plot was tested for accuracy by spot heighting intersections of a drawn profile with the contour lines using an analytical stereoplotter.

5.1.2 Crowsnest Pass stereomodel

This model was sampled twice at grid intervals of 7 and 12 m. Breaklines, hilltops and depressions were also measured. From each grid sampling combined with breaklines, hilltops and depressions, contour lines at 1.0 m interval were interpolated and plotted at 1:3 000 scale. Each contour plot was tested for accuracy as with the Raymond's.

5.2 Results and Discussion

The accuracy of the discrepancies of contour lines may be estimated as follows /1/:

$$\sigma^2_{disc} \approx \sigma^2_{spec} + \sigma^2_{AT} + \sigma^2_{setup} + \sigma^2_{samp}$$

For the Raymond stereomodel,

$$\sigma_{disc} \approx (0.15^2 + 0.07^2 + 0.03^2 + 0.03^2)^{1/2}$$

$$\approx 0.17 \text{ m}$$

For the Crowsnest Pass stereomodel,

$$\sigma_{disc} \approx (0.31^2 + 0.12^2 + 0.05^2 + 0.05^2)^{1/2}$$

$$\approx 0.34 \text{ m}$$

From the RMS's of discrepancies of contour elevations, it is evident that they all meet the required accuracies (see Table 5). From these, it was concluded that the true optimum grid sampling interval for the Raymond stereomodel was 62m, and for the Crowsnest Pass stereomodel was 12m. Based upon these conclusions, sample profiles that were representative of each model's roughness were identified. The range of computed sampling intervals for each profile should include 62 or 12m as the case may be. Differences of computed sampling intervals from the true intervals were computed and summarized in tables 6 and 7.

Differences from 12m are within ± 3m for long and rough profile 7 (see Table 7). For long profiles, RF is the most conservative, while

LINEAR is the most optimistic (see Tables 6 and 7). Percent differences for long profiles are within ± 34% except with program LINEAR.

For short profiles RF is the most conservative while LINEAR is the most optimistic (see Tables 6 and 7). All programs have percent differences exceeding ±50%.

For the Raymond profiles of varying sample spacings, LOGKV and RF have no increasing trends in percent differences (see Table 6). RF is the most conservative, while LINEAR is the most optimistic. Percent differences are within ± 40%.

For the Crowsnest Pass profiles of varying sample spacings, all programs have increasing trends in percent differences (see Table 7). LOGKV is the most conservative, while SPECTRA is the most optimistic. Only program RF has percent differnces within ± 33%.

6. RECOMMENDATIONS

Based upon the results of relative and absolute comparisons, the most suitable program(s) is/are recommended for each sample profile depending on its roughness, length and sampling rate (see Table 8) as follows:

a) Roughness
  - rough (R.F. >5%)
  - flat (R.F. <5%)

Length
  - long (length >1.75 model base)
  - short (length < 1.75 model base)

c) Sampling rate
  - high (Δd <1.0mm at image scale)
  - low (Δd >1.0mm at image scale)

The recommended programs are based upon percent differences within ±50% obtained from the tables referenced and profiles used. If more than one program is recommended, the programs are arranged in increasing degree of optimism. In other words, the first program is the most conservative, while the last is the most optimistic based upon their means of percent differences for more than one profile. In case of contradiction between relative and absolute comparisons the latter prevails. There were no data for (c) and (g).

It can be noted that short profiles can be used only if they are rough and of high sampling rate. For more reliable result, the sample profile shall be as long and of high sampling rate as possible, and shall be truly representative of the model's roughness.

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a) Terrain Aerial Surveys Ltd, Edmonton, Alberta for measuring

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b) North West Survey Group, Edmonton, Alberta for grid sampling using their Wild AC1 analytical stereoplotter and plotting contour lines using their Wild CIP and TA2 table.

c) Stewart, Weir Land Data Inc.,Edmonton, Alberta for testing the accuracy of contour plots using their Wild BC1 analytical stereoplotter.

REFERENCES

/1/ Balce, A.E. 1986. "Determination of Optimum Sampling Interval in Grid Digital Elevation Models (DEM) Data Acquisition", Annual ACSM/ASPRS Convention, March 16-21, 1986, Washington, DC, USA.

/2/ Toomey, M.A.G. (ed.) 1984. Digital Elevation Model Workshop Proceedings, ABSM, October 10-12, 1984, Edmonton, Alberta, Canada.

/3/ Toomey, M.A.G. 1986. "Digital Elevation Models in Alberta", Annual ACSM/ASPRS Convention, March 16-21, 1986, Washington, DC, USA.

Table 1 Long Profiles

	SPECTRA			LOGKV			LINEAR			RF			Mean	R.F.
Prof	Int	Diff	%	Int	Diff	%	Int	Diff	%	Int	Diff	%	(m)	(%)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
9	5	-2	-29	8	1	14	10	3	43	6	-1	-14	7	10.9
7	10	-2	-17	11	-1	-8	15	3	25	10	-2	-17	12	9.3
152	15	3	25	10	-2	-17	11	-1	-8	12	0	0	12	2.3
154	30	10	50	18	-2	-10	20	0	0	11	-9	-45	20	1.6
155	64	9	16	63	8	14	62	7	13	31	-24	-44	55	0.9
153	77	3	4	66	-8	-11	110	36	49	41	-33	-45	74	0.7
RMS			28			13			29			33		
Mean			8			-3			20			-28		

Notes:
(1) Profile identifier
(2), (5), (8), (11) Computed sampling intervals (m)
(14) Mean sampling interval
(3), (6), (9), (12) Differences from the mean
(4), (7), (10), (13) Percent differences with respect to mean
(15) Roughness factor

Table 2 Short Profiles

	SPECTRA			LOGKV			LINEAR			RF			Mean	R.F.
Prof	Int	Diff	%	Int	Diff	%	Int	Diff	%	Int	Diff	%	(m)	(%)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
9-4	6	0	0	6	0	0	8	2	33	4	-2	-33	6	21.1
7-1	6	-2	-25	8	0	0	8	0	0	10	2	25	8	11.3
7-4	12	-2	-14	14	0	0	23	9	64	8	-6	-43	14	10.9
9-1	12	4	50	7	-1	-12	8	0	0	7	-1	-12	8	10.1
9-3	20	6	43	11	-3	-21	20	6	43	7	-7	-50	14	8.8
7-2	12	-3	-20	15	0	0	20	5	33	13	-2	-13	15	8.6
71-1	12	2	20	9	-1	-10	12	2	20	9	-1	-10	10	7.6
7-3	12	0	0	12	0	0	15	3	25	7	-5	-42	12	7.2
71	13	0	0	13	0	0	14	1	8	11	-2	-15	13	5.8
71-2	17	-1	-6	21	3	17	22	4	22	14	-4	-22	18	4.3
152-1	11	3	38	6	-2	-25	8	0	0	7	-1	-12	8	4.0
9-2	20	6	43	12	-2	-14	12	-2	-14	12	-2	-14	14	3.9
1521	16	3	23	12	-1	- 8	16	3	23	7	-6	-46	13	2.6
154-1	21	8	62	12	-1	- 8	13	0	0	7	-6	-46	13	2.0
1541	27	9	50	15	-3	-17	18	0	0	10	-8	-44	18	1.5
154-2	48	12	33	34	-2	- 6	32	-4	-11	29	-7	-19	36	1.1
155-1	48	6	14	42	0	0	59	17	40	18	-24	-57	42	1.0
155-2	96	-13	-12	138	29	27	110	1	1	92	-17	-16	109	0.9
153-1	96	24	33	51	-21	-29	111	39	54	30	-42	-58	72	0.8
153-2	64	-12	-16	108	32	42	84	8	10	46	-30	-40	76	0.8
152-2	96	24	33	70	-2	- 3	83	11	15	38	-34	-47	72	0.7
1531	55	- 9	-14	54	-10	-16	98	34	53	48	-16	-25	64	0.7
RMS			30			16			29			35		
Mean			15			- 4			19			-29		

Notes:

(1) Profile identifier

(2), (5), (8), (11) Computed sampling intervals (m)

(14) Mean sampling interval

(3), (6), (9), (12) Differences from the mean

(4), (7), (10), (13) Percent differences with respect to mean

(15) Roughness factor

Table 3 Raymond Profiles of Varying Sample Spacings

Profile	6 m			9 m			3 m
	Int (m)	Diff	%	Int (m)	Diff	%	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
SPECTRA							
152	15	3	25	16	4	33	12
153	77	3	4	82	8	11	74
154	24	4	20	25	5	25	20
155	64	9	16	72	17	31	55
RMS = 23	Mean = 21						
LOGKV							
152	6	- 6	-50	6	- 6	-50	12
153	53	-21	-28	60	-14	-19	74
154	13	- 7	-35	8	-12	-60	20
155	48	- 7	-13	50	- 5	- 9	55
RMS = 37	Mean = -33						
LINEAR							
152	6	- 6	-50	9	- 3	-25	12
153	79	5	7	87	13	18	74
154	13	- 7	-35	9	-11	-55	20
155	61	6	11	56	1	2	55
RMS = 31	Mean = -16						
RF							
152	11	-1	-8	13	1	8	12
153	37	-37	-50	37	-37	-50	74
154	11	-9	-45	12	- 8	-40	20
155	26	-29	-53	23	-32	-58	55
RMS = 43	Mean = -37						

Notes:
(1) Profile identifier
(2), (5), Computed sampling intervals (m)
(8), Mean sampling interval (14) of Table 1
(3), (6), Differences from (8)
(4), (7) Percent differences with respect to (8)
RMS's and means are of those percent differences (4), (7)

Table 4 Crowsnest Pass Profiles of Varying Sample Spacings

	8 m			12 m			16 m			20 m			4m
Profile	Int (m)	Diff	%	Int (m)	Diff	%	Int (m)	Diff	%	Int (m)	Diff	%	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
SPECTRA													
7	14	2	17	17	5	42	20	8	67	22	10	83	12
9	8	1	14	12	5	71	16	9	129	20	13	186	7
RMS = 94 Mean = 76													
LOGKV													
7	9	- 3	-25	7	-5	-42	6	-6	-50	4	-8	-67	12
9	6	- 1	-14	5	- 2	-29	4	- 3	-43	3	-4	-57	7
RMS = 40 Mean = -41													
LINEAR													
7	8	- 4	-33	12	0	0	16	4	33	20	8	67	12
9	8	1	14	12	5	71	16	9	129	20	13	186	7
RMS = 89 Mean = 58													
RF													
7	11	- 1	- 8	13	1	8	16	4	33	16	4	33	12
9	9	2	29	11	4	57	13	6	86	14	7	100	7
RMS = 54 Mean = 42													

Notes:

(1) Profile identifier

(2), (5), (8), (11) Computed sampling intervals (m)

(14) Mean sampling interval (14) of Table 1

(3), (6), (9), (12) Differences from (14)

(4), (7), (10), (13) Percent differences with respect to (14)

RMS's and means are of those percent differences (4), (7), (10), (13)

Table 5 Discrepancies of Contour Lines

	Discrepancies (m)				
Number	Raymond			Crowsnest Pass	
	62 m	51 m	20 m	12 m	7 m
1	-0.02	-0.05	-0.12	0.62	0.49
2	-0.04	-0.10	-0.18	-0.12	-0.13
3	-0.27	-0.20	-0.28	-0.28	-0.16
4	0.01	-0.03	0.04	0.35	0.54
5	-0.02	-0.02	0.02	0.10	0.02
6	0.01	0.02	0.02	-0.16	-0.21
7	-0.10	-0.10	-0.04	-0.28	-0.17
8	0.16	0.15	0.06	-0.20	-0.07
9	-0.09	-0.06	-0.06	0.72	0.31
10	-0.11	-0.10	-0.08	-0.32	-0.24
11	-0.05	-0.08	0.14	-0.85	-0.53
12	-0.23	0.08	0.00	-0.04	-0.26
13	-0.10	-0.18	-0.04	0.04	-0.46
14	0.04	0.03	-0.02	0.23	-0.22
15	-0.10	-0.08	-0.08	-0.23	-0.16
16	-0.23	-0.04	-	0.12	0.08
17	-0.09	-0.09	-	-0.08	0.00
18	-0.14	0.10	-0.10	-0.25	-0.32
19	-0.06	-0.11	-0.15	-0.37	-0.48
20	-0.09	-0.09	-0.12	0.01	0.09
21	-0.08	-0.10	-0.10	0.01	-0.04
22	-0.04	-0.03	0.04	0.05	0.00
23	-	-	-	0.30	0.16
24	-0.08	-0.12	-0.19	-0.09	-0.04
25	-	-	-	-0.42	-0.68
26	-	-	-	0.12	-0.17
27	-	-	-	-0.48	-0.63
28	-	-	-	-0.05	-0.50
RMS	0.12	0.10	0.11	0.32	0.32

Table 6 Differences of Sampling Intervals from 62 m

	R.F.	SPECTRA			LOGKV			LINEAR			RF		
Profile	(%)	Int	Diff	%	Int	Diff	%	Int	Diff	%	Int	Diff	%
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
153	0.7	77	15	24	66	4	6	110	48	77	41	-21	-34
153-1	0.8	96	34	55	51	-11	-18	111	49	79	30	-32	-52
153-2	0.8	64	2	3	108	46	74	84	22	36	46	-16	-26
153 1	0.7	55	-7	-11	54	- 8	-13	98	36	58	48	-14	-23
Mean				16			14			58			-34
153 (6)		77	15	24	53	- 9	-14	79	17	27	37	-25	-40
(9)		82	20	32	60	- 2	- 3	87	25	40	37	-25	-40
Mean				28			- 8			34			-40

Notes:
(1) Profile identifier
(2) Roughness factor
(3), (6), (9), (12) Computed sampling intervals (m)
(4), (7), (10), (13) Differences from 62 m
(5), (8), (11), (14) Percent differences with respect to 62 m

Table 7 Differences of Sampling Intervals from 12 m

	R.F.	SPECTRA			LOGKV			LINEAR			RF		
Profile	(%)	Int	Diff	%	Int	Diff	%	Int	Diff	%	Int	Diff	%
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
7	9.3	10	-2	-17	11	- 1	- 8	15	3	25	10	- 2	-17
7-4	10.9	12	0	0	14	2	17	23	11	92	8	- 4	-33
9-1	10.1	12	0	0	7	- 5	-42	8	- 4	-33	7	- 5	-42
9-3	8.8	20	8	67	11	- 1	- 8	20	8	67	7	- 5	-42
7-2	8.6	12	0	0	15	3	25	20	8	67	13	1	8
71-1	7.6	12	0	0	9	- 3	-25	12	0	0	9	- 3	-25
7-3	7.2	12	0	0	12	0	0	15	3	25	7	- 5	-42
71	5.8	13	1	8	13	1	8	14	2	17	11	- 1	- 8
9-2	3.9	20	8	67	12	0	0	12	0	0	12	0	0
Mean				18			- 3			29			-23
7(8)		14	2	17	9	- 3	-25	8	- 4	-33	11	- 1	- 8
(12)		17	5	42	7	- 5	-42	12	0	0	13	1	8
(16)		20	8	67	6	- 6	-50	16	4	33	16	4	33
(20)		22	10	83	4	- 8	-67	20	8	67	16	4	33
9(12)		12	0	0	5	- 7	-58	12	0	0	11	- 1	- 8
(16)		16	4	33	4	- 8	-67	16	4	33	13	1	8
(20)		20	8	67	3	- 9	-75	20	8	67	14	2	17
Mean				44			-55			24			12

Notes:

(1) Profile identifier

(2) Roughness factor

(3), (6), (9), (12) Computed sampling intervals (m)

(4), (7), (10), (13) Differences from 12 m

(5), (8), (11), (14) Percent differences with respect to 12 m

Table 8 Program Guidelines

Profile Attributes	Most Suitable Program(s)	Table(s) Referenced	Profile(s) Used
a) Rough, long, high	LOGKV, RF, SPECTRA OR LINEAR	1 4 7	7, 9 7(8), 9(8) 7, 7(8)
b) Rough, long, low	RF	7	7(12, 16, 20), 9(12, 16, 20)
c) Rough, short, low	-	-	-
d) Rough, short, high	RF or LOGKV	7	7-2, 7-3, 7-4, 9-1, 9-2, 9-3, 71, 71-1
e) Flat, long high	RF, LOGKV or SPECTRA	6	153
f) Flat, long, low	RF, LOGKV SPECTRA or LINEAR	6	153(6, 9)
g) Flat, short low	-	-	-
h) Flat, short high	None suitable	-	-

N.L.C. - B.N.C.
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